# ABERRATIONS

## Order of Aberration

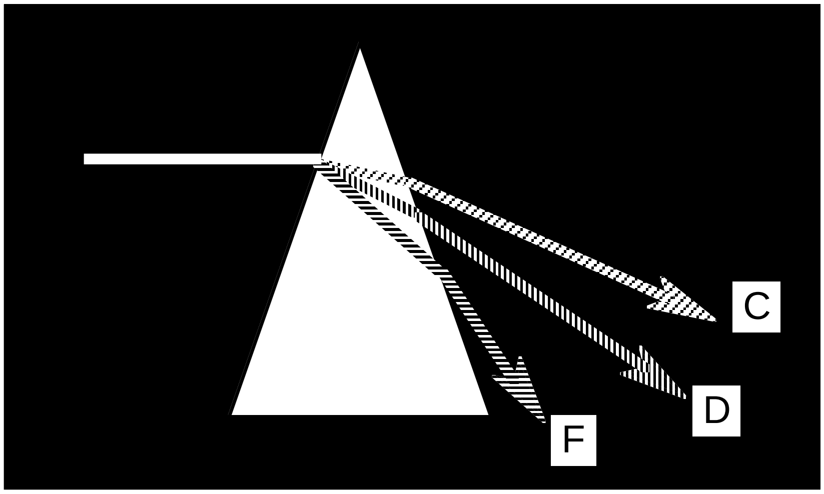
In paraxial theory we've assumed that light rays move along nearly parallel to the optical axis and that a given medium has the same optical index for all light. In fact, neither of these is exactly true. The general MacLaurin's series for the sine function is as follows:

$$\sin\theta=\theta-\theta^3/3!+\theta^5/5!-\theta^7/7!+\ldots$$

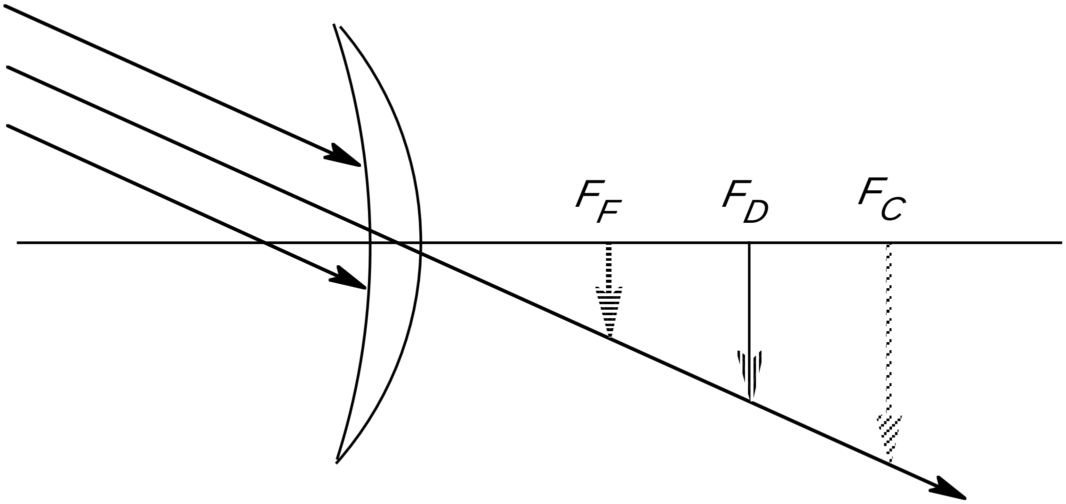
In the paraxial theory we took only the first term of this expansion, $\sin\theta\cong\theta$. By adding higher order terms we can develop a more accurate theory. Discrepancies from the paraxial theory are termed <u>aberrations</u>. Aberrations are classified according to the power of $\theta$ in the expansion above, i.e. first order, third order, fifth order, etc.

## First Order Aberration-Chromatic Aberration

The easiest aberration to understand is chromatic aberration since it arises in first order--paraxial--theory. It stems from the fact that the index of refraction of a material is different for light of different wavelengths. This is well known from the fact that white light striking a prism breaks up into colored rays. As result, the image formed by a lens will be slightly different for each color component of light.



The letters in the diagram above pertain to certain spectral bands. The D band is a yellow band in the sodium spectrum very near the peak sensitivity of the human visual system. The C and F lines are red and green-blue lines from the hydrogen spectrum.



Suppose we have a lens like that in the diagram above refracting collimated white light. Different images will form for each spectral line, as shown, due to chromatic aberration. Lateral chromatic aberration is the difference in size of the images due to the chromatism of the lens. Longitudinal chromating aberration is the difference in position of the images due to the chromatism of the lens.

Chromatism of a material is characterized most commonly by its nu-value or constringence defined as

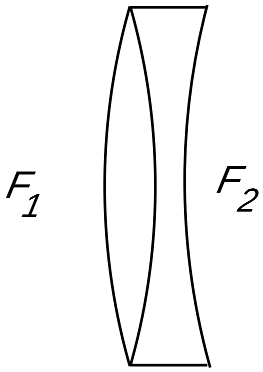
$$\nu=(n_D-1)/(n_F-n_C).$$

The subscripts on the indices of refraction apply to the spectral bands. The **larger** the constringence, the **less** the chromatism.

If we have a thin lens of surface radii $r_1$ and $r_2$, the difference in focal powers for the F and C lines may be written

$$F_F-F_C=n_F(r_1-r_2)-n_C(r_1-r_2)=(n_F-n_C)(r_1-r_2)=[(n_F-n_C)/(n_D-1)]F_D=F_D/\nu,$$
$$F_F-F_C=F_D/\nu \qquad (1)$$

So how do we beat out chromatic aberration? In a single lens like a spectacle lens we pick a lens material with the highest constringence possible. In an optical instrument, however, we can use an achromatic doublet like that shown below. It is formed from two thin lenses of powers $F_1$ and $F_2$.



The lens must have a particular power $F$ so

$$F=F_{1D}+F_{2D}. \tag{2}$$

From (1), the chromatic differences in power just cancel out if

$$F_{1D}/\nu_1=-F_{2D}/\nu_2. \tag{3}$$

Using (2) and (3) we can design an achromatic doublet of given power.

Example: Design an achromatic doublet of +5.00D power using lenses of materials with constringences of 30 and 60.

Solution: From (2), $+5=F_{1D}+F_{2D}$. From (3), $F_{1D}/30=-F_{2D}/60$ or $2F_{1D}=-F_{2D}$. This gives two equations in two unknowns. Eliminating $F_{2D}$ between the first and last of these equations,

$$+5=F_{1D}-2F_{1D}=-F_{1D} \text{ or } F_{1D}=-5.00\text{D}.$$

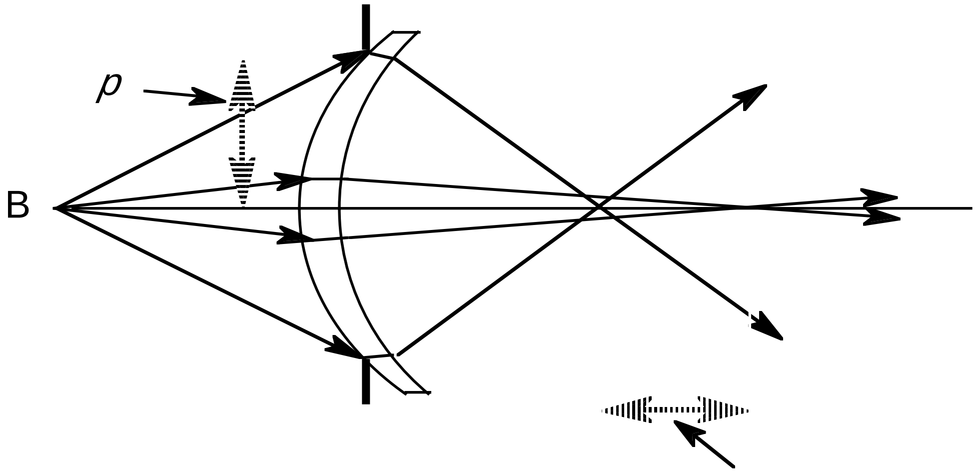
Thus $F_{2D}=-2\text{x}5=-10.00\text{D}$. Thus the lens of constringence 30 should have -5.00D power and the lens of constringence 60 should have +10.00D power.

# Third Order Aberrations

For wide angles, rays don't all focus nicely at a point, so no precise image is formed. The deviations from focus may be described by third order theory in terms of five Seidel aberrations. These are monochromatic aberrations characterized by certain geometric effects.

## Spherical Aberration

Spherical aberration is an axial aberration and so may be completely described by meridional rays. It occurs as shown in the diagram below.



lateral spherical aberration

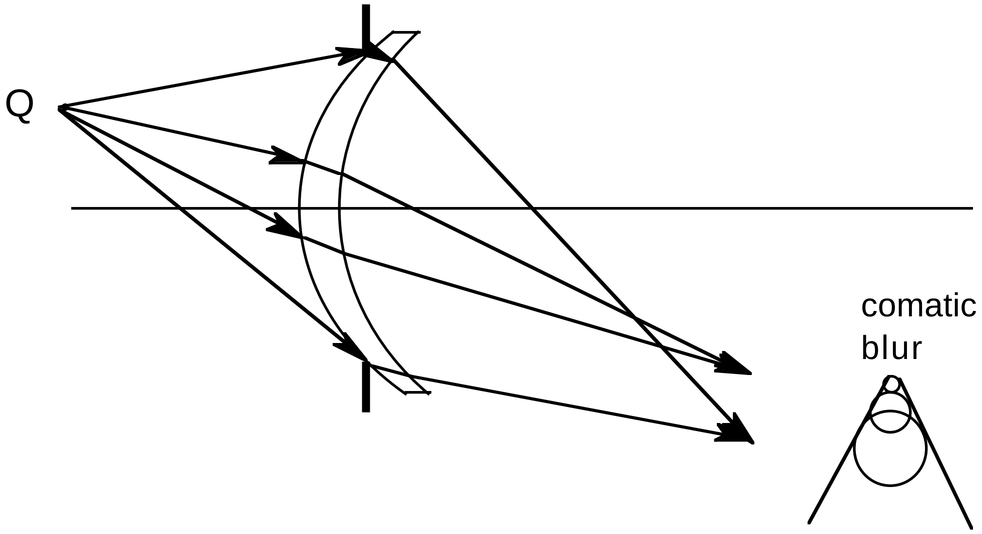
As the diagram shows, light rays in the paraxial region focus at a different point than light rays going through the periphery of the lens. The distance between the two foci is the lateral spherical aberration. In the case above the peripheral rays are more bent than the paraxial rays, which is called positive spherical aberration. If the peripheral rays were bent and the peripheral focus were farther from the lens, we would have negative spherical aberration.

The lateral spherical aberration is proportional to the square of the entrance pupil diameter or lateral spherical aberration$\propto p^2$.

Spherical aberration can be minimized by "bending" lenses to obtain the optimum lens form, by using a small aperture stop, or by using a parabolic lens.

# Coma

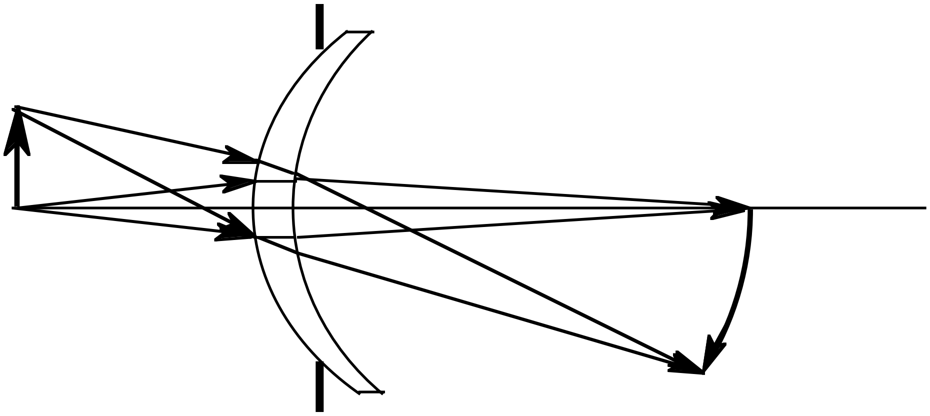
Like spherical aberration, coma is a wide angle aberration, but it applies only to off axial rays. Its origin is shown below. The comatic blur figure is assymetric and "comet shaped", hence the name coma.



Coma may be eliminated in a lens by appropriate choice of curves. A wide angle aberration, it may be minimized by using a small aperture stop.

## Curvature of Field

Curvature of field is another oblique aberration. In curvature of field a plane object is sharply imaged, but on a curved surface, as shown below.

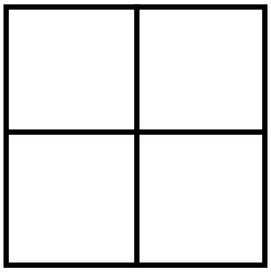


The surface on which the image is formed is called the Petzval surface.

Curvature of field is a small aperture aberration, as shown, and narrowing the aperture stop will have no effect on it. In may be handled by bending the lens, relocating the stop position, or curving the receiving the screen.

# Distortion

Another oblique small angle aberration is distortion. In distortion the object is sharply imaged but doesn't retain its shape. There are two kinds of distortion, barrel distortion and pincushion distortion, so named because of their effect on a square grid target.


object

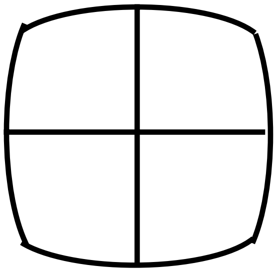


image with barrel distortion

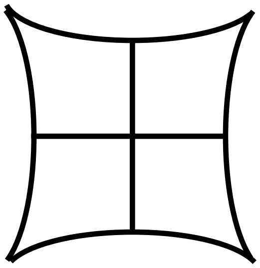
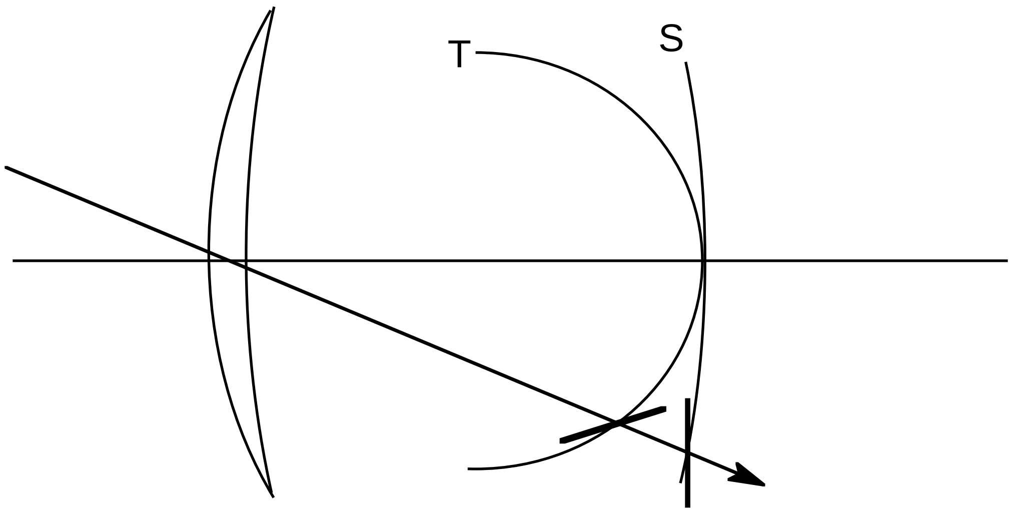


image with pincushion distortion

Distortion can be dealt with through placement of the aperture or by using lens systems, e.g. a so-called orthoscopic doublet.

# Marginal Astigmatism

Marginal astigmatism or oblique astigmastism is another small angle oblique third order aberration. Even though a lens is a spherical lens, oblique rays form astigmatic images with separated line foci. In the following diagram the loci of the sagittal line focus, the line focus in the plane of the page, is labelled by S and the loci of the tangential line focus, the line focus perpendicular to the page, is labelled by T.



Because of the appearance of the loci, this is sometimes called the "teacup and saucer" aberration. Note that the two loci touch on the axis.

Marginal astigmatism is very important in ophthalmic optics. It is the origin of the astigmatism induced by tilting a spherical lens. And it is the aberration modern corrected curve designs seek to eliminate. It can be controlled by lens bending or with appropriate lens combinations in optical instruments.

As the aperture is opened wider and wider, the astigmatic figure of marginal astigmatism becomes the typical comatic figure.

# Aberration Zoology

The table below summarizes in tabular form the taxonomy of the aberrations discussed.

| | **chromatic aberration** | **spherical aberration** | **coma** | **curvature of field** | **distortion** | **marginal astigmatism** |
|---|---|---|---|---|---|---|
| **order** | 1st | 3rd | 3rd | 3rd | 3rd | 3rd |
| **aperture dependence** | no | yes | yes | no | no | no |
| **blur form** | circular blur, colored fringes | circular | comatic figure | no blur | no blur | focal lines |
| **remedy** | achromatic doublet, high ν | bend lens, small aperture, parabolic lens | bend lens, small aperture | bend lens, bend image plane | orthoscopic doublet, stop placement | bend lens |
| **key words** | constringence, ν-value | | comet shaped | Petzval surface | barrel, pincushion | teacup & saucer |
| **corrected in spectacles** | somewhat | no | no | yes | no | yes |