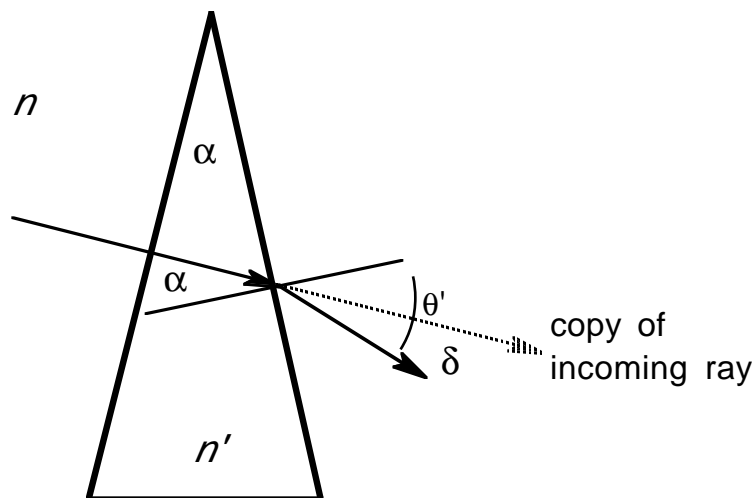# OPHTHALMIC PRISMS

Thin Prisms

In ophthalmic optics, only thin prisms, those of small apical angle, are important and only rays incident nearly normally are considered. That simplifies the problem of tracing rays through the prism a lot. The diagram below shows such a prism. We want to calculate the deviation, δ, of a ray passing through the prism

Let's simplify the calculation by having the ray incident normally on the first face of the prism. It is not, then deviated by the first face and strikes the second face such that it makes an angle α with the normal. It leaves the prism at an angle θ' with the normal. From Snell's law,

$$n\sin\alpha = n'\sin\theta' \text{ or } n'\alpha \cong n\theta'.$$

(5)

But the angle of deviation δ is related to θ' by

$$\theta' = \alpha + \delta.$$

(6)

Eliminating θ' from (5) and (6),

$$\delta = [(n'/n) - 1]\alpha.$$

(7)

Even though (7) was derived for a nearly normally incident ray, it is true for any ray incident nearly normally. That means that all such rays are deviated through exactly the same angle. If the rays come from an object B, as in the diagram below, an observer sees them as if they came from an image B', displaced vertically from B but not displaced longitudinally.
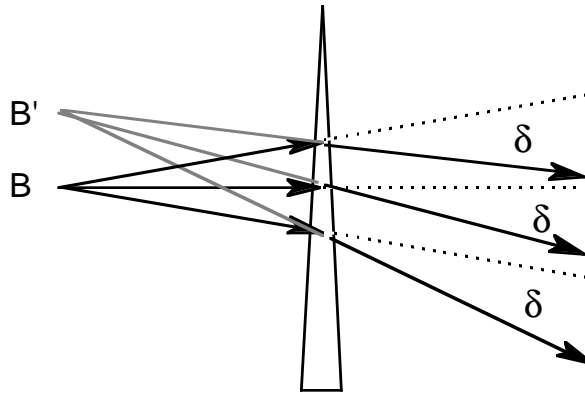
Ophthalmic prisms are thin prisms. They are measured in <u>prism diopters</u> of prism power. A prism which causes a deviation of a ray of light of 1cm per meter travelled by the ray has one prism diopter of prism power. The diagram below shows how a one diopter prism deviates a ray of light.
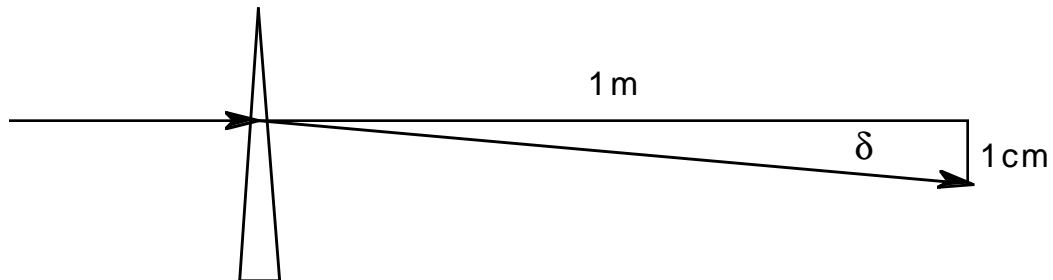
From the diagram, the angle of deviation is δ(radians)=1cm/1m=0.01. If the prism has Δ prism diopters of power, it moves the beam Δ centimeters for every meter travelled and $\delta=\Delta$cm/1m=0.01Δ or

$$\Delta=100\delta(radians)=100(n'-1)\alpha(radians)$$

(8)

since n=1 for prisms in air.

Example: What is the power of a crown glass prism of apical angle 4°?

Solution: α=4°=(π/180°)x4°=0.070 radians and n'=1.523. Plugging into (8) gives Δ=100(1.523-1)(0.070)=3.65prism diopters$\cong$4 p.d.

Textbooks make allusion to a unit called the <u>centrad</u>. The centrad *per se* is of purely historical interest and for all practical purposes a centrad is the same as a prism diopter.

Prisms are incorporated into spectacles in order to compensate binocular problem like squint. The prism is usually used to move the image to the direction in which the eye is aimed. The actual problem usually faced by the optometrist is to **avoid** inadvertantly giving prism.

The orintation of a prism in ophthalmic work is usually given by the following abbreviations:

BU   base up
BD   base down
BI   base in (to the right for the right eye, left for the left eye)
BO   base out (to the left for the right eye, right for the left eye)
BR   base right
BL   base left

Superposition of Thin Prisms

If a prism is placed base down before the eye, the image appears to move up, the amount of image movement being proportional to the angle of deviation. A 5 p.d. BD prism moves the image upward 5 "units". Likewise, a 3 p.d. BL psim would move the image to the right 5 "units". What, then, would happen if the 5 p.d. BD and 3 p.d. BL prisms were superimposed. Clearly, the image would move up and to the right, 5 "units" up and 3 "units" to the right. But that sounds like vector addition, right? And in fact, one can apply vector addition to ophthalmic prisms.
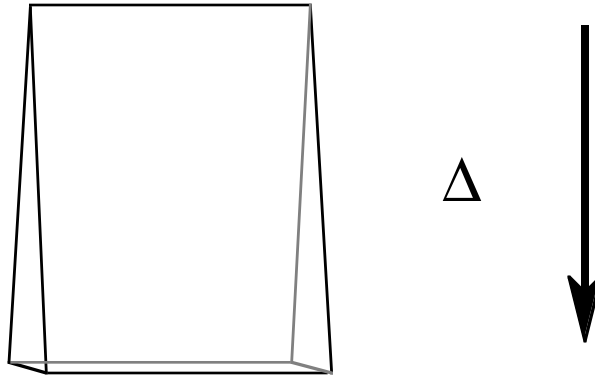
As shown in the diagram above, an ophthalmic prism may be represented as a vector with the magnitude of the prism and direction from apex-to-base. It would have been nicer if the opposite direction since the image moves toward the apex and the prism sort of points that way, but that just isn't how it's done.

Superimposed prisms may be combined by just adding the corresponding vectors and they may be resolved into components, just like any other vectors.

Example: The apex-to-base line of a 6p.d. prism makes an angle of 60° with the x-axis. What is the prism power in the horizontal and vertical meridians?

Solution: Draw the vector representation of the prism. From trigonometry $\Delta_x = 6\cos 60° = 3$p.d., $\Delta_y = 6\sin 60° = 5.2$p.d.
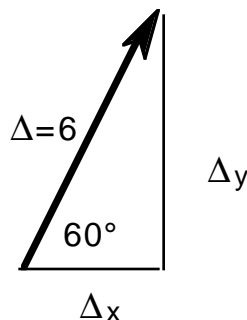
The whole vector could be written in component form as $\Delta = (3.0,\ 5.2)$.

Example: What prism is equivalent to a 5p.d. base @ 45° prism superimposed on a 3p.d. base down prism?

Solution: Resolve each prism into components. The vector corresponding to the first prism is $\Delta_1=(5\cos45°,5\sin45°)=(3.5,3.5)$. The vector corresponding to the second prism is $\Delta_2=(0,-3)$. The total prism is the vector sum,
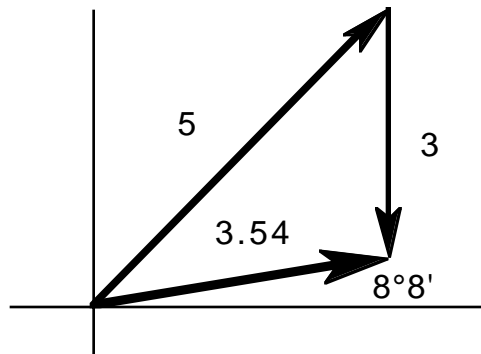
$$\Delta=\Delta_1+\Delta_2=(3.5+0,3.5-3)=(3.5,0.5).$$

The magnitude of the resultant prism is

$$\Delta=\sqrt{[(3.5)^2+(0.5)^2]}=3.54\text{p.d.}$$

and the angle is
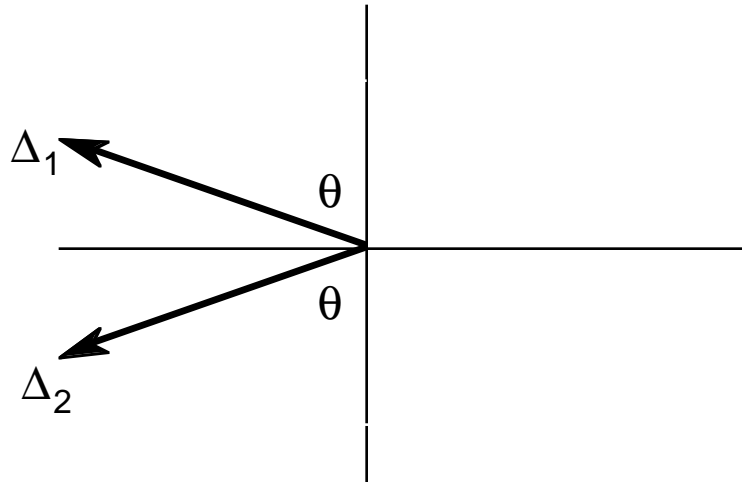
$$\theta=\text{arc }\tan(0.5/3.5)=8°8'.$$

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The Risley Prism

An interesting example of the vector addition of prisms is the Risley prism used in phorometry. Schematically this consists of two prisms of identical magnitude oriented schematically as shown below:

The vertical components of the two prisms cancel out since they are geared to move in tandom, and the two horizontal components, which are identical, add to give a resultant of $2\Delta\sin\theta$ (and BL in the drawing above).

Prismatic Effects in Lenses

We can think of a lens as a stack of prisms, each with slightly different power. Not surprisingly, then, extra-axial points behanve as if they had a prism power as well as a lens power. We can calculate that prism power from the diagram below.
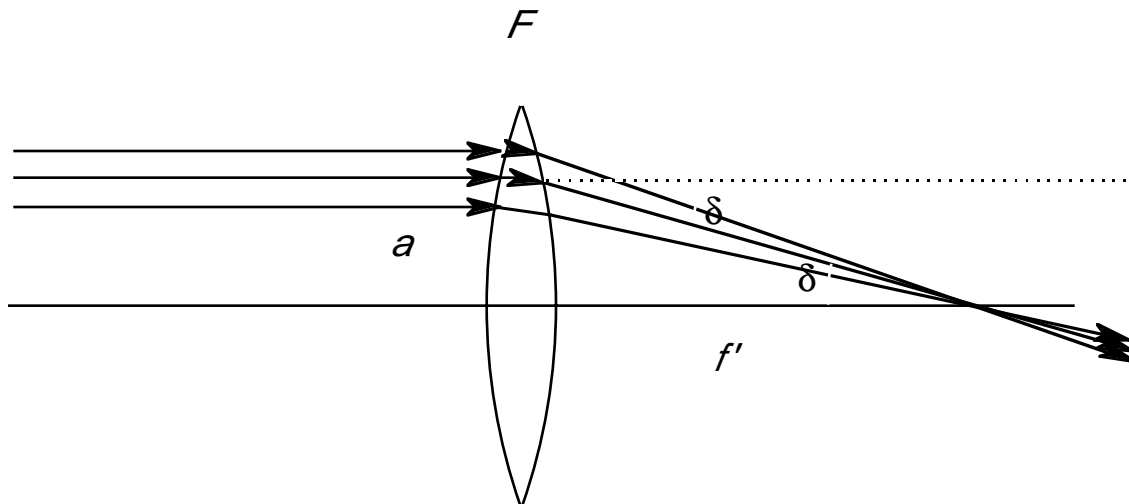
Consider a parallel bundle of rays striking a lens of power F a distance a from the optical axis. Rays converge to the secondary focal point. The ray bundle is rotated through an angle δ where $\delta=-a/f'=-aF$. To get the prismatic deviation, remember that prism power Δ is one hundred times the angle of deviation in radians so

$$\Delta=100\delta=-100aF.$$

This is the famous Prentice's Rule. If we give a in centimeters we can drop the factor of 100 and write simply

$$\Delta=-a(\text{cm})F.$$

Example: What is the prism 2 centimeters above the optic axis for a +3.00D lens?

Solution: $\Delta=-(2\text{cm})(+3.00\text{D})=-6\text{p.d.}$, 6p.d. base down.

In laying out a lens for edging, an optician can achieve small amounts of prism by decentering a lens a distance d. That decentration is equal in magnitude by opposite in direction to a so $d=-c$.

Thickness of Ophthalmic Prisms

Since lens blanks come in a fixed size, e.g. 65mm round, often there is not enough lens to do the necessary decentration. In such a case it is necessary to grind on the back surface such that there is an angle between base and apex, using appropriate tools to grind in the appropriate curves. The result is then like a circular piece trephined from a much larger blank.
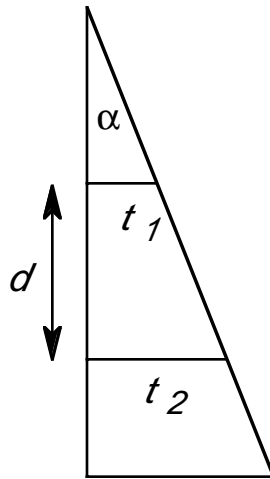
The diagram shows a thin prism with apical angle α and two cross-sections of the prism of thicknesses t_1 and t_2 separated by distance d. From striaghtforward geometry, remembering that α is a small angle,

$$\alpha = (t_2 - t_1)/d.$$

But $\Delta = 100(n\text{-}1)\alpha$ so the difference in thicknesses may be written,

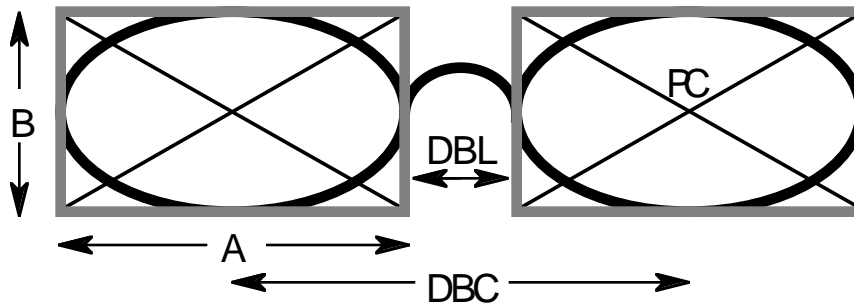$$t_2 - t_1 = d\alpha = d\Delta/[100(n\text{-}1)]$$

Example: A 4 p.d. prism is surfaced onto a Crown glass blank 65mm in diameter. If the thinnest edge of the prism is 2.2mm thick, how thick is the thickest edge?

Solution: From the equation above,

$$t_2 = t_1 + d\Delta/[100(n\text{-}1)] = 2.2\text{mm} + (65\text{mm})(4\text{p.d.})/[100(1.523\text{-}1)] = 7.2\text{mm}.$$

Reference Points of a Spectacle Frame

Since 1962, the boxing system has been adopted by the OMA (Optical manufacturer's Association) to specify frame dimensions. That system draws a square or "box" around each lens and designates certain reference points and distances with respect to that box. The following diagram indicates the boxing terminology. Below the diagram are the definition of nomenclature associated with the boxing system.

B	vertical measurement
A	horizontal measurement or eye size
DBL	distance between lenses or bridge size
A-B	pattern of the lens
PC	pattern center, the geometric center formed by the intersection of the diagonals of the box
OC	optical center of the lens, the point at which there is no prism
PD	pupillary distance; there are two types:
	(1) patient PD, the distance between the optical axes of the patient's eyes measured in the spectacle plane;
	(2) frame PD, the distance between geometric centers (DBC) where DBC=DBL+A.
MRP	major reference point, the point on the lens having the lens prescription with propoer prism, usually the same as the OC
ED	effective diameter, twice the distance from the geometrical center fo the peak of the lens bevel that is farthest from the geometrical center. Minimum blank size=ED+2x(decentration).
level of MRP	distance of MRP from line A
LD	lateral decentration of MRP, the distance from GC that MRP must be moved

Frames are specified as AŸDBL, i.e. 50Ÿ20 where this means A=50mm, DBL=20mm.

The usual clinical problem is this: for a given frame, Rx, and patient PD, what must be the lateral decentration? Here are the steps in the solution:

        (1)   Determine MRP
        (2)   Determine PD
        (3)   Determine A and DBL

Assuming we want the eyes symmetric in the primary position of gaze, we get the lateral decentration from

$$LD=[(\text{frame PD})-(\text{patient PD})]/2.$$

If LD<0 decentration is out. If LD>0 decentration is in.

Example: The frame specification is 52Ÿ22 and the patient PD is 65mm. Find the decentration necessary to have no prism in front of the eyes in the primary position of gaze.

Solution: The frame PD=52+22=74 so LD=(74-65)/2=4.5 in.

If we want the total decentration, we have to add in the decentration necessary to get to the MRP from the OC.

Example: In the previous example, the MRP was reached by decentering lenses 10 out and 5 down. Find total decentration.

Solution: total decentration=LD+10out+5down=14.5out+5down.

Some Practical Prescription Considerations

Prism blurs vision if it's in excess of 3p.d., so only rarely, if ever, should more than that be prescribed. To keep prism down, it is usually divided between the eyes.   Instead of prescribing

<div align="center">

O.D. 4BI, 2BU
O.S. no prism

</div>

give

<div align="center">

O.D. 2BI, 1BU
O.S. 2BI, 1BD

</div>

Relative BI prism requires less convergence, a boon for exophores, a curse for esophores.

Relative BO prism requires more convergence, a curse for exophores, a boon for esophores.

Even small amounts of vertical prism, 1 to 2 prism diopters, can produce dissociation and severe symptoms.

Generally prism is prescribed in opposite directions for the two eyes. **Overall** prism is sometimes prescribed in cases of muscle problems such as those associated with hyperthyroidism so that the eyes won't have to move to a field of gaze in which there is diplopia.

A pair of misguided ophthalmologists a few years ago proposed using overall prism to move images to the good part of a low vision patient's retina. This "prism scanning technique" was taken quite seriously for a while, but the flaw was eventually recognized. The flaw, of course, is that the patient's eye will just follow the image, relocating it on the bad part of his retina, if that is his wont.

Prism and Convergence

Because prism is used to make the eye move or to compensate for the fact that the eye doesn't move, it makes sense to talk about prism in terms of the amount of rotation a given prism requires the to make. For a remote object, we have the situation shown below:
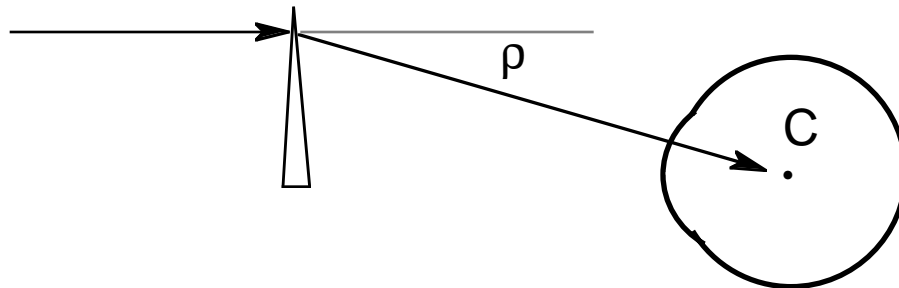
 Here the eye must rotate through an angle $\rho=\delta$ to see the object, where δ is the angle of deviation of the prism. For a near object, however, the situation is as below:
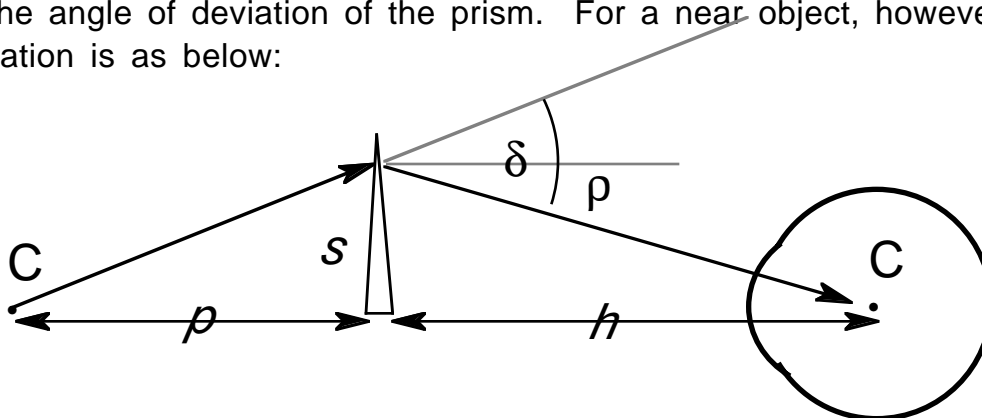
Here, clearly, $\rho<\delta$. From the geometry, $s/p=\delta-\rho$ and $s/h=\rho$. Dividing these equations gives $h/p=(\delta-\rho)/\rho$ or

$$\rho=\delta p/(p+h).$$

In terms of eye rotation, the effective power of a prism of power Δ held a distance h from the eye used in viewing an object a distance p from the prism is $\Delta \Rightarrow [p/(p+h)]\Delta$.

Example: A 5p.d. prism placed 15mm from the eye is used to view an object 40cm from the eye. What is the effective value of this prism?

Solution: effective prism=[40cm/(40cm+1.5cm)](5p.d.)=4.8p.d.

Clearly, in this realistic example the correction is negligible. In fact it always is.

In convergence the eyes must look through points on the spectacle lenses more nasal than the optical center and deal with the prism found there. The amount of prism depends on patient PD, lens vertex distance, and the distance of the object of regard from the spectacle plane. We calculate the approximate prism encountered from the diagram below.
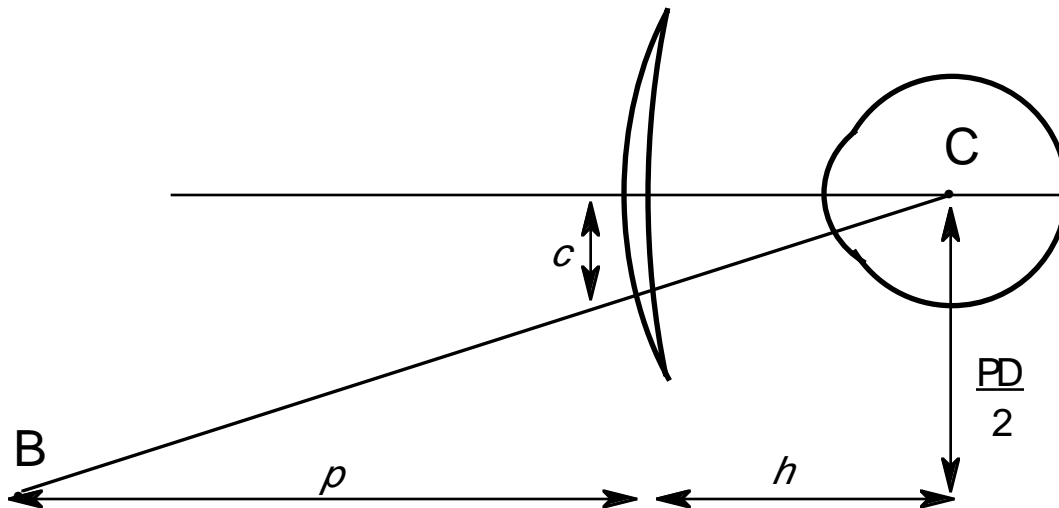
Assume the eye turns the same amount to see B with the lens present as it would with it absent. (It doesn't, but the difference is negligible.) The point on the lens cut by the line of sight is a distance c from the optical center. From the geometry of the diagram, using similar triangles $c/h=(PD/2)/(p+h)$, so

$$c=(PD/2)[h/(p+h)].$$

If all distances are given in centimeters,

$$\Delta=cF=hF(PD/2)/(p+h).$$

Example: A -2.50DS myope reads at 40cm from the spectacle plane. How much less must he converge with glasses than without, assuming his glasses are 25mm from the center of rotation of his eye and his PD is 70mm?

Solution: D=(2.5cm)(-2.5D)(7.0cm/2)/(40cm+2.5cm)=0.51p.d. Thus there is 0.53p.d. BI prism for each eye or a total of only one prism diopter diminished convergence necessary.

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