

# LUMINANCE PROBLEMS

More difficult problems are indicated with an asterisk.

1. Plot the cosine function in polar coordinates. What is the shape of the plot?
2. A plane Lambert radiator is a square 2 mm on a side with luminance 400 cd/m<sup>2</sup>. What is the intensity of the radiator viewed along the line (a) normal to its surface, and (b) 30° from the normal to its surface?
3. The intensity of a Lambert radiator is 80 cd measured along a line 25° with the surface normal. What is the intensity measured along a line 80° with the surface normal?
4. An illuminance meter is 12 ft from a circular Lambert radiator one inch in diameter with 100 ft-cd luminance. The normal to the meter head makes an angle of 45° with the line connecting meter and radiator and the normal to the radiator makes an angle of 30° with that line. What is the meter reading?
- 5.\* A plane 50 cd/m<sup>2</sup> Lambert radiator of area 10 cm<sup>2</sup> is placed on the floor 3 m from and parallel to a wall. What is the illuminance (a) along the baseboard, and (b) one meter above the floor, and (c) three meters above the floor?
- 6.\* Show that if a small Lambert radiator is parallel to a small receiving plane, the illuminance of the receiving plane is proportional to  $\cos^4\theta$ , where  $\theta$  is the angle between the normal to the source and the line connecting source and receiving planes.
7. An illuminance meter with a head of one square centimeter area is placed with its head parallel to and 50 cm above the center of a circular Lambert radiator of 220 lux luminance and 25 cm radius. What is the meter reading?

8. A radiator has a luminance of 15 ft-cd. What is its luminance in (a) ft-Lamberts, (b) lux, and (c) apostilbs?
9. A one apostilb disk shaped Lambert radiator has a radius of one meter. A small illuminance meter is placed a distance  $x$  directly above the center of the disk and parallel to it. Plot the meter reading as a function of  $x$  using the exact solution. On the same graph plot the infinite plane and inverse square asymptotic. Indicate on your graph the range of distances for which neither asymptotic form is accurate to within 10%.
10. An illuminance meter is held with its head near and parallel to a large Lambert radiator of 20 lux luminance. What is the meter reading?
- 11.\* A flat ring shaped Lambert radiator has 20 cm inside diameter and 100 cm outside diameter. Its luminance is 1200 lux. An illuminance meter is placed with its head parallel to the ring and 100 cm above its center. What is the meter reading?
- 12.\* In visual field testing, clinicians sometimes use a unit of retinal sensitivity, the decibel (dB). The decibel is related logarithmically to threshold target luminance. If 1 asb corresponds to 40 dB and 10,000 asb corresponds to 0 dB, how many dB is equivalent to (a) 1000 asb, (b) 10 asb, (c) 500 asb. Conversely, how many asb correspond to (d) 20 dB, (e) 5 dB. [Hint: decibels are related to asb by an equation of the form  $\text{dB} = c_1 + c_2 \log(\text{asb})$ , where  $c_1$  and  $c_2$  are constants. Solve for  $c_1$  and  $c_2$  and use the resulting equation to answer (a)-(e).]
- 13.\* A forty-two year old patient complains that he sees fine working at his desk, but has trouble reading the menu in a fancy restaurant. Why do you suppose this is?

- 14.\* The illuminance at an axial point on a plane parallel to and a distance  $R$  from a disk of luminance  $L$  and radius  $a$  is

$$E = 2\pi LR^2 \int_0^a \frac{\rho d\rho}{(R^2 + \rho^2)^2}.$$

Evaluate the integral in this equation.

- 15.\* When placed at a large distance  $r$  from a luminous polygon of area  $S$  with its head parallel to that surface, a light meter reads illuminance  $E$ . What will its reading be when it is placed very close to, parallel to, and near the center of the surface?
16. A light meter is placed parallel to and above the edge of a semi-infinite plane of 100 asb luminance. What is the meter reading?
- 17.\* A Lambert radiator is shaped like a square with sides two meters long. A light meter held three centimeters above and parallel to the center of the square reads 300 lm/sq m. What will the meter read if it is moved 150 m above the center of the square?

## ANSWERS to SELECTED PROBLEMS

1. The plot is a circle.
2. (a)  $1.6 \times 10^{-3}$  cd; (b)  $1.39 \times 10^{-3}$  cd
3. 15.3 cd
4.  $2.32 \times 10^{-3}$  lm/ft<sup>2</sup>
5. (b)  $4.5 \times 10^{-3}$  lm/m<sup>2</sup>
7. 138 lm/m<sup>2</sup>
8. (a) 47 ft-L

10. 63 lm/m<sup>2</sup>
11. 717 lm/m<sup>2</sup>. The trick here is to calculate the illuminance due to a 100 cm diameter disk and subtract the illuminance due to a 20 cm diameter disk.
12. The conversion equation is  $\text{dB} = 40 - 10 \log(\text{asb})$  so the answers are (a) 10 dB; (b) 30 dB; (c) 13 dB; (d) 100 asb; (e) 3162 asb.
13. Being a fancy restaurant, the light is probably dim and in dim light visual acuity goes down. The patient attempts to compensate by moving material closer to his eyes--but finds that his presbyopia prevents his focusing it clearly.
14. Hint: integrate using change of variables, the new variable being  $u = (R^2 + \rho^2)$ .
15.  $\pi E r^2 / S$
16. 50 lm/m<sup>2</sup>
17. 0.017 lm/sq m