

# WAVES PROBLEMS

More difficult problems are indicated with an asterisk.

1. If  $x$  is position and  $t$  time, which of the following functions could be a wave function? What is the speed and direction of travel of each *bona fide* wave function?
  - (a)  $y = x + 6t$
  - (b)  $y = x[\sin(3t)]$
  - (c)  $y = \log x - \log(2t)$
  - (d)  $y = x^2 - 6xt + 9t^2$
  - (e)  $y = 4x^2 - 9t^2$
  - (f)  $y = 9e^x e^{4t}$
  - (g)  $y = 4 \tan(x/t)$
2. Plot the wave function  $y(x, t) = \exp[-(x - 3t)^2]$  at time  $t = 0$  and  $t = 1$ . In what direction does the wave move? If  $x$  is in meters and  $t$  is in seconds, what is the speed of movement of the wave?
3. A sine wave has the form  $y(x, t) = 14 \sin[2\pi(x/13 - t/12) + 6]$ . What are the amplitude, wavelength, frequency, period, spatial frequency, speed of propagation, propagation number, angular frequency, and phase factor of the wave?
4. Find the sum of the two wave functions  $y_1(x, t) = 10 \sin[2\pi(x/18 - t/4) + 3]$  and  $y_2(x, t) = \sin[2\pi(x/18 - t/4) + 9]$ .
- 5.\* Plot (on graph paper) the functions  $y_1 = 8 \sin(5\pi x)$  and  $y_2 = 3 \sin(5\pi x - \pi/3)$ . Add the two functions point by point to obtain  $y = y_1 + y_2$ . From your graph estimate the amplitude and phase of  $y$ . Confirm your result by adding the two wave functions vectorially.

6.\* The Fourier series for a function takes the form

$$f(x) = (4/\pi)[\sin x + (1/3)\sin 3x + (1/5)\sin 5x + \dots]$$

Plot (on graph paper) the first four or more terms in the expansion and add them point by point. Add terms in the expansion until you can guess at the form of  $f(x)$ .

## ANSWERS to SELECTED PROBLEMS

1. (a)  $v = -6$ , (d)  $v = 3$ , (f),  $v = -4$ . None of the others are wave functions.

2.  $v = 3\text{m/sec}$ .

|    |                    |        |
|----|--------------------|--------|
| 3. | amplitude          | 14.000 |
|    | wavelength         | 13.000 |
|    | period             | 12.000 |
|    | frequency          | 0.083  |
|    | spatial frequency  | 0.077  |
|    | speed              | 1.083  |
|    | propagation number | 0.483  |
|    | angular frequency  | 0.524  |
|    | phase factor       | 6.000  |

4. These waves correspond to vectors 10 at 3 rad or (-9.90, 1.41), and 1 at 9 rad or (-0.91, 0.41). The sum of these vectors is (-10.81, 1.82). This corresponds to a vector magnitude 10.96. Since the x-component is negative and the y-component is positive, the vector is in the second quadrant and the angle of the vector is  $\arctan[1.82/(-10.81)] = \arctan(-0.1684) = -0.1684 + \pi = 2.9732$  rad. The corresponding wave function is then

$$y(x, t) = 10.96 \sin[2\pi(x/18 - t/4) + 2.97].$$